MATHEMATICS 9th Science Group

7/18/2020

Chapter 7.

LINEAR EQUATIONS AND INEQUALITIES

Contents

Radical Equation

When the variable in an equation occurs under a radical, the equation is called a radical equation.
For example,

$$\sqrt{x-3}-7=0$$

Linear Equation

A linear equation in one unknown variable x is an equation of the form $ax+b=0$, where $a,b\in R$ and $a\neq 0$

A solution to a linear equation is any replacement or substitution for the variable x that makes the statement true. Two linear equations are said to be equivalent if they have exactly the same solution.
For example, $x+1=0,\ \ 2x+5=-1$

EXERCISE 7.1

Q#1) Solve the following equations.

(i). $\frac{2}{3}x-\frac{1}{2}x=x+\frac{1}{6}$

Solution: As given $\frac{2}{3}x-\frac{1}{2}x=x+\frac{1}{6}$

Multiply by 6(LCM) on both sides

$$6\times\frac{2}{3}x-6\times\frac{1}{2}x=6\times x+6\times\frac{1}{6}$$
$$4x-3x=6x+1$$
$$x=6x+1$$
$$1=x-6x$$
$$1=-5x$$
$$x=-\frac{1}{5}$$

Check:

$$\frac{2}{3}x-\frac{1}{2}x=x+\frac{1}{6}$$

Put $x=-\frac{1}{5}$

$$\frac{2}{3}\left(-\frac{1}{5}\right)-\frac{1}{2}\left(-\frac{1}{5}\right)=\left(-\frac{1}{5}\right)+\frac{1}{6}$$
$$-\frac{2}{15}+\frac{1}{10}=-\frac{1}{5}+\frac{1}{6}$$

Multiply by 30(LCM) on both sides

$$30\times\left(-\frac{2}{15}\right)+30\times\left(\frac{1}{10}\right)$$
$$=30\times\left(-\frac{1}{5}\right)+30\times\left(\frac{1}{6}\right)$$
$$-4+3=-6+5$$
$$-1=-1\text{(which is true)}$$

Since $x=-\frac{1}{5}$ satisfy the given equation, therefore, the solution set is $\left\{-\frac{1}{5}\right\}$ i.e. $S.S=\left\{-\frac{1}{5}\right\}$

(ii). $\frac{x-3}{3}-\frac{x-2}{2}=-1$

Solution: As given $\frac{x-3}{3}-\frac{x-2}{2}=-1$

Multiply by 6(LCM) on both sides

$$6\times\left(\frac{x-3}{3}\right)-6\times\left(\frac{x-2}{2}\right)=6\times(-1)$$
$$2(x-3)-3(x-2)=-6$$
$$2x-6-3x+6=-6$$
$$-x=-6$$
$$x=6$$

Check:

$$\frac{x-3}{3}-\frac{x-2}{2}=-1$$

Put $x=6$

$$\frac{(6)-3}{3}-\frac{(6)-2}{2}=-1$$
$$\frac{3}{3}-\frac{4}{2}=-1$$
$$1-2=-1$$
$$-1=-1\text{ (which is true)}$$

Since $x=6$ satisfy the given equation, therefore, the solution set is $\{6\}$ i.e. $S.S=\{6\}$

(ii). $\frac{1}{2}\left(x-\frac{1}{6}\right)+\frac{2}{3}=\frac{5}{6}+\frac{1}{3}\left(\frac{1}{2}-3x\right)$

Solution:

As given $\frac{1}{2}\left(x-\frac{1}{6}\right)+\frac{2}{3}=\frac{5}{6}+\frac{1}{3}\left(\frac{1}{2}-3x\right)$

$$\frac{1}{2}x-\frac{1}{12}+\frac{2}{3}=\frac{5}{6}+\frac{1}{6}-x$$

Multiply by 12(LCM) on both sides

$$12\times\left(\frac{1}{2}x\right)-12\times\left(\frac{1}{12}\right)$$
$$+12\times\left(\frac{2}{3}\right)=12\times\left(\frac{5}{6}\right)+12\times\left(\frac{1}{6}\right)-12\times(x)$$
$$6x-1+8=10+2-12x$$
$$6x+7=12-12x$$
$$6x+12x=12-7$$
$$18x=5$$
$$x=\frac{5}{18}$$

Check:

$$\frac{1}{2}\left(x-\frac{1}{6}\right)+\frac{2}{3}=\frac{5}{6}+\frac{1}{3}\left(\frac{1}{2}-3x\right)$$
$$\frac{1}{2}x-\frac{1}{12}+\frac{2}{3}=\frac{5}{6}+\frac{1}{6}-x$$

Put $x=\frac{5}{18}$

$$\frac{1}{2}\left(\frac{5}{18}\right)-\frac{1}{12}+\frac{2}{3}=\frac{5}{6}+\frac{1}{6}-\left(\frac{5}{18}\right)$$
$$\frac{5}{36}-\frac{1}{12}+\frac{2}{3}=\frac{5}{6}+\frac{1}{6}-\frac{5}{18}$$

Multiply by 36(LCM) on both sides

$$36\times\left(\frac{5}{36}\right)-36\times\left(\frac{1}{12}\right)$$
$$+36\times\left(\frac{2}{3}\right)=36\times\left(\frac{5}{6}\right)+36\times\left(\frac{1}{6}\right)-36\times\left(\frac{5}{18}\right)$$

$$5-3+24=30+6-10$$
$$-3+29=36-10$$
$$26=26 \quad \text{(which is true)}$$

Since $x=\frac{5}{18}$ satisfy the given equation, therefore, the solution set is $\left\{\frac{5}{18}\right\}$ i.e. $S.S=\left\{\frac{5}{18}\right\}$

(iv). $\boldsymbol{x+\frac{1}{3}=2\left(x-\frac{2}{3}\right)-6x}$

Solution: As given $x+\frac{1}{3}=2\left(x-\frac{2}{3}\right)-6x$

$$x+\frac{1}{3}=2x-\frac{4}{3}-6x$$

Multiply by 3(LCM) on both sides

$$3\times(x)+3\times\left(\frac{1}{3}\right)$$
$$=3\times(2x)-3\times\left(\frac{4}{3}\right)-3\times(6x)$$
$$3x+1=6x-4-18x$$
$$3x+1=-4-12x$$
$$3x+12=-4-1$$
$$15x=-5$$
$$x=-\frac{5}{15}$$
$$x=-\frac{1}{3}$$

Check:

$$x+\frac{1}{3}=2\left(x-\frac{2}{3}\right)-6x$$
$$x+\frac{1}{3}=2x-\frac{4}{3}-6x$$

Put $x=-\frac{1}{3}$

$$\left(-\frac{1}{3}\right)+\frac{1}{3}=2\left(-\frac{1}{3}\right)-\frac{4}{3}-6\left(-\frac{1}{3}\right)$$
$$-\frac{1}{3}+\frac{1}{3}=-\frac{2}{3}-\frac{4}{3}+2$$

Multiply by 3(LCM) on both sides

$$-1+1=-2-4+6$$
$$0=0 \text{ (which is true)}$$

Since $x=-\frac{1}{3}$ satisfy the given equation, therefore, the solution set is $\left\{-\frac{1}{3}\right\}$ i.e. $S.S=\left\{-\frac{1}{3}\right\}$

(v) $\boldsymbol{\frac{5(x-3)}{6}-x=1-\frac{x}{9}}$

Solution: As given $\frac{5(x-3)}{6}-x=1-\frac{x}{9}$

Multiply by 18 (LCM) on both sides

$$18\times\left(\frac{5(x-3)}{6}\right)-18\times(x)$$
$$=18\times(1)-18\times\left(\frac{x}{9}\right)$$
$$3(5x-15)-18x=18-2x$$
$$15x-45-18x=18-2x$$
$$-45-3x=18-2x$$
$$-3x+2x=18+45$$
$$-x=63$$
$$x=-63$$

Check:

$$\frac{5(x-3)}{6}-x=1-\frac{x}{9}$$

Put $x=-63$

$$\frac{5(-63-3)}{6}-(-63)=1-\frac{(-63)}{9}$$
$$\frac{5(-66)}{6}+63=1+\frac{63}{9}$$
$$-55+63=1+7$$
$$8=8 \text{ (which is true)}$$

Since $x=-63$ satisfy the given equation, therefore, the solution set is $\{-63\}$ i.e. $S.S=\{-63\}$

(vi). $\boldsymbol{\frac{x}{3x-6}=2-\frac{2x}{x-2},\ x\neq 2}$

Solution: As given $\frac{x}{3x-6}=2-\frac{2x}{x-2}$

$$\frac{x}{3x-6}=\frac{2(x-2)-2x}{x-2}$$
$$\frac{x}{3x-6}=\frac{2x-4-2x}{x-2}$$
$$\frac{x}{3x-6}=\frac{-4}{x-2}$$
$$x(x-2)=-4(3x-6)$$
$$x^2-2x=-12x+24$$
$$x^2-2x+12x-24=0$$
$$x(x-2)+12(x-2)=0$$
$$(x-2)(x+12)=0$$

That is $x=2,-12$

Since it is given that $x\neq 2$, therefore, we ignore $x=2$ and just check $x=-12$ for the solution set.

Check:

$$\frac{x}{3x-6}=2-\frac{2x}{x-2}$$

Put $x=-12$

$$\frac{(-12)}{3(-12)-6}=2-\frac{2(-12)}{(-12)-2}$$
$$\frac{-12}{-36-6}=2+\frac{24}{-12-2}$$
$$\frac{-12}{-42}=2+\frac{24}{-14}$$
$$\frac{2}{7}=\frac{28-24}{14}$$
$$\frac{2}{7}=\frac{4}{14}$$
$$\frac{2}{7}=\frac{2}{7} \quad \text{(which is true)}$$

Since $x=-12$ satisfy the given equation, therefore, the solution set is $\{-12\}$ i.e. $S.S=\{-12\}$

(vii). $\frac{2x}{2x+5}=\frac{2}{3}-\frac{5}{4x+10}$, $x\neq\frac{6}{2}$

Solution: As given $\frac{2x}{2x+5}=\frac{2}{3}-\frac{5}{4x+10}$

$$\frac{2x}{2x+5}=\frac{2(4x+10)-15}{3(4x+10)}$$
$$\frac{2x}{2x+5}=\frac{8x+20-15}{12x+30}$$
$$\frac{2x}{2x+5}=\frac{8x+5}{12x+30}$$
$$2x(12x+30)=(8x+5)(2x+5)$$
$$24x^2+60x=16x^2+40x+10x+25$$
$$24x^2+60x=16x^2+50x+25$$
$$24x^2+60x-16x^2-50x-25=0$$
$$8x^2+10x-25=0$$
$$8x^2+20x-10x-25=0$$
$$4x(2x+5)-5(2x+5)=0$$
$$(2x+5)(4x-5)=0$$

That is $x=-\frac{5}{2},\frac{5}{4}$

Since it is given that $x\neq-\frac{5}{2}$, therefore, we ignore $x=-\frac{5}{2}$ and just check $x=\frac{5}{4}$ for the solution set.

Check:

$$\frac{2x}{2x+5}=\frac{2}{3}-\frac{5x}{4x+10}$$

Put $x=\frac{5}{4}$

$$\frac{2\left(\frac{5}{4}\right)}{2\left(\frac{5}{4}\right)+5}=\frac{2}{3}-\frac{5}{4\left(\frac{5}{4}\right)+10}$$
$$\frac{\frac{5}{2}}{\frac{5}{2}+5}=\frac{2}{3}-\frac{5}{5+10}$$
$$\frac{\frac{5}{2}}{\frac{5+10}{2}}=\frac{2}{3}-\frac{5}{15}$$
$$\frac{5}{15}=\frac{2}{3}-\frac{1}{3}$$
$$\frac{1}{3}=\frac{1}{3}\quad\text{(which is true)}$$

Since $x=\frac{5}{4}$ satisfy the given equation, therefore, the solution set is $\left\{\frac{5}{4}\right\}$ i.e. $S.S=\left\{\frac{5}{4}\right\}$

(viii). $\frac{2x}{x-1}+\frac{1}{3}=\frac{5}{6}+\frac{2}{x-1}$, $x\neq 1$

Solution: As given $\frac{2x}{x-1}+\frac{1}{3}=\frac{5}{6}+\frac{2}{x-1}$

$$\frac{2x}{x-1}+\frac{1}{3}=\frac{5}{6}+\frac{2}{x-1}$$
$$\frac{3(2x)+(x-1)}{3(x-1)}=\frac{5(x-1)+2(6)}{6(x-1)}$$
$$\frac{6x+x-1}{3(x-1)}=\frac{5x-5+12}{6(x-1)}$$
$$\frac{7x-1}{3(x-1)}=\frac{5x+7}{6(x-1)}$$
$$(7x-1)6(x-1)=3(x-1)(5x+7)$$
$$6(7x-1)(x-1)-3(x-1)(5x+7)=0$$
$$3(x-1)[2(7x-1)-(5x+7)]=0$$
$$3(x-1)(14x-2-5x-7)=0$$
$$3(x-1)(9x-9)=0$$
$$3(x-1)9(x-1)=0$$
$$27(x-1)^2=0$$

Which implies that

$(x-1)=0$ gives that $x=1$ which is not possible (given $x\neq 1$)

therefore, the solution set is {} i.e. $S.S=\{\}$

(ix). $\frac{2}{x^2-1}-\frac{1}{x+1}=\frac{1}{x+1}$, $x\neq\pm1$

Solution: As given $\frac{2}{x^2-1}-\frac{1}{x+1}=\frac{1}{x+1}$

$$\frac{2}{(x+1)(x-1)}-\frac{1}{x+1}=\frac{1}{x+1}$$
$$\frac{2-(x-1)}{(x+1)(x-1)}=\frac{1}{x+1}$$
$$\frac{2-x+1}{(x+1)(x-1)}=\frac{1}{x+1}$$
$$\frac{3-x}{(x+1)(x-1)}=\frac{1}{x+1}$$
$$(3-x)(x+1)=(x+1)(x-1)$$
$$(3-x)(x+1)-(x+1)(x-1)=0$$
$$(x+1)[(3-x)-(x-1)]=0$$
$$(x+1)(3-x-x+1)=0$$
$$(x+1)(4-2x)=0$$

That is $x=-1,2$

Since it is given that $x\neq\pm1$, therefore, we ignore $x=-1$ and just check $x=2$ for the solution set.

Check:

$$\frac{2}{x^2-1}-\frac{1}{x+1}=\frac{1}{x+1}$$

Put $x=2$

$$\frac{2}{(2)^2-1}-\frac{1}{(2)+1}=\frac{1}{(2)+1}$$
$$\frac{2}{4-1}-\frac{1}{3}=\frac{1}{3}$$
$$\frac{2}{3}-\frac{1}{3}=\frac{1}{3}$$
$$\frac{1}{3}=\frac{1}{3}\quad\text{(which is true)}$$

Since $x=2$ satisfy the given equation, therefore, the solution set is {2} i.e. $S.S=\{2\}$

(x). $\frac{2}{3x+6}=\frac{1}{6}-\frac{1}{2x+4}$, $x\neq-2$

Solution: As given $\frac{2}{3x+6}=\frac{1}{6}-\frac{1}{2x+4}$

$$\frac{2}{3x+6}=\frac{1(2x+4)-6}{6(2x+4)}$$

$$\frac{2}{3x+6}=\frac{2x+4-6}{6(2x+4)}$$
$$\frac{2}{3x+6}=\frac{2x-2}{6(2x+4)}$$
$$\frac{2}{3(x+2)}=\frac{2(x-1)}{6(2(x+2))}$$
$$\frac{2}{3(x+2)}=\frac{(x-1)}{6(x+2)}$$
$$12(x+2)=3(x+2)(x-1)$$
$$12(x+2)-3(x+2)(x-1)=0$$

$$3(x+2)[4-x+1)]=0$$
$$3(x+2)(5-x)=0$$

That is $x=-2,5$

Since it is given that $x\neq -2$, therefore, we ignore $x=-2$ and just check $x=5$ for the solution set.

Check:

$$\frac{2}{3x+6}=\frac{1}{6}-\frac{1}{2x+4}$$

Put $x=5$

$$\frac{2}{3(5)+6}=\frac{1}{6}-\frac{1}{2(5)+4}$$
$$\frac{2}{15+6}=\frac{1}{6}-\frac{1}{10+4}$$
$$\frac{2}{21}=\frac{1}{6}-\frac{1}{14}$$
$$\frac{2}{21}=\frac{7-3}{42}$$
$$\frac{2}{21}=\frac{2}{42}$$

$\frac{2}{21}=\frac{2}{21}$ (which is true)

Since $x=5$ satisfy the given equation, therefore, the solution set is $\{5\}$ i.e. $S.S=\{5\}$

Q#2) Solve each equation and check for extraneous solution if any.

(i). $\sqrt{3x+4}=2$

Solution: As given $\sqrt{3x+4}=2$

On squaring, we get

$$\left(\sqrt{3x+4}\right)^2=(2)^2$$
$$3x+4=4$$
$$3x=0$$
$$x=0$$

Check:

$$\sqrt{3x+4}=2$$

Put $x=0$

$$\sqrt{3(0)+4}=2$$
$$\sqrt{0+4}=2$$

$2=2$ (which is true)

Since $x=0$ satisfy the given equation, therefore, the solution set is $\{0\}$ i.e. $S.S=\{0\}$

(ii). $\sqrt[3]{2x-4}-2=0$

Solution: As given $\sqrt[3]{2x-4}-2=0$

$$\sqrt[3]{2x-4}=2$$

Taking cube on both sides

$$\left(\sqrt[3]{2x-4}\right)^3=(2)^3$$
$$2x-4=8$$
$$2x=8+4$$
$$2x=12$$
$$x=6$$

Check:

$$\sqrt[3]{2x-4}-2=0$$

Put $x=6$

$$\sqrt[3]{2(6)-4}-2=0$$
$$\sqrt[3]{12-4}-2=0$$
$$\sqrt[3]{8}-2=0$$
$$2-2=0$$

$0=0$ (which is true)

Since $x=6$ satisfy the given equation, therefore, the solution set is $\{6\}$ i.e. $S.S=\{6\}$

(ii). $\sqrt{x-3}-7=0$

Solution: As given $\sqrt{x-3}-7=0$

$$\sqrt{x-3}=7$$

Taking square on both sides

$$\left(\sqrt{x-3}\right)^2=(7)^2$$
$$x-3=49$$
$$x=49+3$$
$$x=52$$

Check:

$$\sqrt{x-3}-7=0$$

Put $x=52$

$$\sqrt{52-3}-7=0$$
$$\sqrt{49}-7=0$$
$$7-7=0$$

$0=0$ (which is true)

Since $x=52$ satisfy the given equation, therefore, the solution set is $\{52\}$ i.e. $S.S=\{52\}$

(iii). $2\sqrt{t+4}=5$

Solution: As given $2\sqrt{t+4}=5$

Taking saquare on both sides

$$\left(2\sqrt{t+4}\right)^2=(5)^2$$
$$4(t+4)=25$$
$$4t+16=25$$
$$4t=25-16$$
$$t=\frac{9}{4}$$

Check:

$$2\sqrt{t+4} = 5$$

Put $t = \frac{9}{2}$

$$2\sqrt{\frac{9}{4}+4} = 5$$

$$2\sqrt{\frac{9+16}{4}} = 5$$

$$2\sqrt{\frac{25}{4}} = 5$$

$$2\left(\frac{5}{2}\right) = 5$$

$$5 = 5 \quad \text{(which is true)}$$

Since $t = \frac{9}{4}$ satisfy the given equation, therefore, the solution set is $\left\{\frac{9}{4}\right\}$ i.e. $S.S = \left\{\frac{9}{4}\right\}$

(v). $\sqrt[3]{2x+3} = \sqrt[3]{x-2}$

Solution: As given $\sqrt[3]{2x+3} = \sqrt[3]{x-2}$

Taking cube on both sides

$$\left(\sqrt[3]{2x+3}\right)^3 = \left(\sqrt[3]{x-2}\right)^3$$

$$2x+3 = x-2$$

$$2x - x = -2 - 3$$

$$x = -5$$

Check:

$$\sqrt[3]{2x+3} = \sqrt[3]{x-2}$$

Put $x = -5$

$$\sqrt[3]{2(-5)+3} = \sqrt[3]{(-5)-2}$$

$$\sqrt[3]{-10+3} = \sqrt[3]{-5-2}$$

$$\sqrt[3]{-7} = \sqrt[3]{-7}$$

Taking cube root, we have

$$-7 = -7 \text{ (which is true)}$$

Since $x = -5$ satisfy the given equation, therefore, the solution set is $\{-5\}$ i.e. $S.S = \{-5\}$

(v). $\sqrt[3]{2-t} = \sqrt[3]{2t-28}$

Solution: As given $\sqrt[3]{2-t} = \sqrt[3]{2t-28}$

Taking cube on both sides

$$\left(\sqrt[3]{2-t}\right)^3 = \left(\sqrt[3]{2t-28}\right)^3$$

$$2 - t = 2t - 28$$

$$2 + 28 = 2t + t$$

$$30 = 3t$$

$$t = 10$$

Check:

$$\sqrt[3]{2-t} = \sqrt[3]{2t-28}$$

Put $t = 10$

$$\sqrt[3]{2-10} = \sqrt[3]{2(10)-28}$$

$$\sqrt[3]{-8} = \sqrt[3]{20-28}$$

$$\sqrt[3]{-8} = \sqrt[3]{-8}$$

Taking cube root, we have

$$-8 = -8 \text{ (which is true)}$$

Since t= 10 satisfy the given equation, therefore, the solution set is $\{10\}$ i.e. $S.S = \{10\}$

(viii). $\sqrt{\frac{x+1}{2x+5}} = 2, x \neq -\frac{5}{2}$

Solution: As given $\sqrt{\frac{x+1}{2x+5}} = 2$

Taking square on both sides

$$\left(\sqrt{\frac{x+1}{2x+5}}\right)^2 = (2)^2$$

$$\frac{x+1}{2x+5} = 4$$

$$x + 1 = 4(2x+5)$$

$$x + 1 = 8x + 20$$

$$1 - 20 = 8x - x$$

$$-19 = 7x$$

$$x = -\frac{19}{7}$$

Check:

$$\sqrt{\frac{x+1}{2x+5}} = 2$$

Put $x = -\frac{19}{7}$

$$\sqrt{\frac{\left(-\frac{19}{7}\right)+1}{2\left(-\frac{19}{7}\right)+5}} = 2$$

$$\sqrt{\frac{\frac{-19+7}{7}}{\frac{-38+35}{7}}} = 2$$

$$\sqrt{\frac{\frac{-12}{7}}{\frac{-3}{7}}} = 2$$

$$\sqrt{\frac{12}{3}} = 2$$

$$\sqrt{4} = 2$$

$$2 = 2 \text{ (which is true)}$$

Since $x = -\frac{19}{7}$ satisfy the given equation, therefore, the solution set is $\left\{-\frac{19}{7}\right\}$ i.e. $S.S = \left\{-\frac{19}{7}\right\}$

Absolute Value

The Absolute value of real number $'a'$ is denoted by $|a|$, is defined as

$$|a| = \begin{cases} a & if\ a \geq 0 \\ -a & if\ a < 0 \end{cases}$$

For example, $|6| = 6$, $|-5| = -(-5) = 5$

$$|0| = 0$$

Some Properties of Absolute value

If $a, b \in R$, then

(i). $|a| \geq 0$

(ii). $|-a| = |a|$

(iii). $|ab| = |a||b|$

(iv). $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$, $|b| \neq 0$

EXERCISE 7.2

Q#1) **1. Identify the following statements as True or False.**

(i) $|\ x\ | = 0$ has only one solution. ⋯T⋯

(ii) All absolute value equations have two solutions. F ⋯

(iii) The equation $|\ x\ | = 2$ is equivalent to $x = 2\ or\ x = -2$. ⋯T⋯⋯

(iv) The equation $|\ x - 4\ | = -4$ has no solution. ⋯F

(v) The equation $|\ 2x - 3\ | = 5$ is equivalent to $2x - 3 = 5\ or\ 2x + 3 = 5$. ⋯F ⋯

Q#2) Solve for x

(i). $|3x - 5| = 4$

Sol: As given $|3x - 5| = 4$

By definition, we have

$3x - 5 = 4$ or $3x - 5 = -4$

$3x = 4 + 5$ or $3x = -4 + 5$

$3x = 9$ or $3x = 1$

$x = 3$ or $x = \frac{1}{3}$

Check:

$$|3x - 5| = 4 \ldots (1)$$

Put $x = 3$, in (1)

$$|3(3) - 5| = 4$$
$$|9 - 5| = 4$$
$$|4| = 4$$
$$4 = 4 \text{ (which is true)}$$

Put $x = \frac{1}{3}$, in (1)

$$\left|3\left(\frac{1}{3}\right) - 5\right| = 4$$
$$|1 - 5| = 4$$
$$|-4| = 4$$
$$4 = 4 \text{ (which is true)}$$

Since $x = 3, \frac{1}{3}$ satisfy the given equation, therefore, the solution set is $\left\{3, \frac{1}{3}\right\}$ i.e. $S.S = \left\{3, \frac{1}{3}\right\}$

(ii). $\mathbf{\frac{1}{2}|3x + 2| - 4 = 11}$

Solution: As given $\frac{1}{2}|3x + 2| - 4 = 11$

$$\frac{1}{2}|3x + 2| = 11 + 4$$
$$\frac{1}{2}|3x + 2| = 15$$
$$|3x + 2| = 30$$

By definition, we have

$3x + 2 = 30$ or $3x + 2 = -30$

$3x = 30 - 2$ or $3x = -30 - 2$

$3x = 28$ or $3x = -\frac{32}{3}$

$x = \frac{28}{3}$ or $x = -\frac{32}{3}$

Check:

$$\frac{1}{2}|3x + 2| - 4 = 11 \ldots (1)$$

Put $x = \frac{28}{3}$, in (1)

$$\frac{1}{2}\left|3\left(\frac{28}{3}\right) + 2\right| - 4 = 11$$
$$\frac{1}{2}|28 + 2| - 4 = 11$$
$$\frac{1}{2}|30| - 4 = 11$$
$$\frac{1}{2}(30) - 4 = 11$$
$$15 - 4 = 11$$
$$11 = 11 \text{ (which is true)}$$

Put $x = -\frac{32}{3}$, in (1)

$$\frac{1}{2}\left|3\left(-\frac{32}{3}\right) + 2\right| - 4 = 11$$
$$\frac{1}{2}|-32 + 2| - 4 = 11$$
$$\frac{1}{2}|-30| - 4 = 11$$
$$\frac{1}{2}(30) - 4 = 11$$
$$15 - 4 = 11$$
$$11 = 11 \text{ (which is true)}$$

Since $x = \frac{28}{3}, -\frac{32}{3}$ satisfy the given equation, therefore, the solution set is $\left\{\frac{28}{3}, -\frac{32}{3}\right\}$ i.e. $S.S = \left\{\frac{28}{3}, -\frac{32}{3}\right\}$

(iii). $\mathbf{|2x + 5| = 11}$

Solution: As given $|2x + 5| = 11$

By definition, we have

$2x + 5 = 11$ or $2x + 5 = -11$

$2x = 11 - 5$ or $2x = -11 - 5$

$2x = 6$ or $2x = -16$

$x = 3$ or $x = -8$

Check:

$$|2x+5| = 11 \ldots(1)$$

Put $x = 3$, in (1)

$$|2(3)+5| = 11$$
$$|6+5| = 11$$
$$|11| = 11$$
$$11 = 11 \text{ (which is true)}$$

Put $x = -8$, in (1)

$$|2(-8)+5| = 11$$
$$|-16+5| = 11$$
$$|-11| = 11$$
$$11 = 11 \text{ (which is true)}$$

Since $x = 3, -8$ satisfy the given equation, therefore, the solution set is $\{3, -8\}$ i.e. $S.S = \{3, -8\}$

(iii). $|3+2x| = |6x-7|$

Solution: As given $|3+2x| = |6x-7|$

$$\frac{|3+2x|}{|6x-7|} = 1$$
$$\left|\frac{3+2x}{6x-7}\right| = 1$$

By definition, we have

$\frac{3+2x}{6x-7} = 1$ or $\frac{3+2x}{6x-7} = -1$

$3+2x = 6x-7$ or $3+2x = -6x+7$

$3+7 = 6x-2x$ or $2x+6x = 7-3$

$10 = 4x$ or $8x = 4$

$x = \frac{5}{2}$ or $x = \frac{1}{2}$

Check:

$$|3+2x| = |6x-7| \ldots(1)$$

Put $x = \frac{5}{2}$, in (1)

$$\left|3+2\left(\frac{5}{2}\right)\right| = \left|6\left(\frac{5}{2}\right)-7\right|$$
$$|3+5| = |15-7|$$
$$|8| = |8|$$
$$8 = 8 \text{ (which is true)}$$

Put $x = \frac{1}{2}$, in (1)

$$\left|3+2\left(\frac{1}{2}\right)\right| = \left|6\left(\frac{1}{2}\right)-7\right|$$
$$|3+1| = |3-7|$$
$$|4| = |-4|$$
$$4 = 4 \text{ (which is true)}$$

Since $x = \frac{5}{2}, \frac{1}{2}$ satisfy the given equation, therefore, the solution set is $\left\{\frac{5}{2}, \frac{1}{2}\right\}$ i.e. $S.S = \left\{\frac{5}{2}, \frac{1}{2}\right\}$

(v). $|x+2| - 3 = 5 - |x+2|$

Solution: As given $|x+2| - 3 = 5 - |x+2|$

$$|x+2| + |x+2| = 5+3$$
$$2|x+2| = 8$$
$$|x+2| = 4$$

By definition, we have

$x+2 = 4$ or $x+2 = -4$

$x = 4-2$ or $x = -4-2$

$x = 2$ or $x = -6$

Check:

$$|x+2| - 3 = 5 - |x+2| \ldots(1)$$

Put $x = 2$, in (1)

$$|2+2| - 3 = 5 - |2+2|$$
$$|4| - 3 = 5 - |4|$$
$$4 - 3 = 5 - 4$$
$$1 = 1 \text{ (which is true)}$$

Put $x = -6$, in (1)

$$|-6+2| - 3 = 5 - |-6+2|$$
$$|-4| - 3 = 5 - |-4|$$
$$4 - 3 = 5 - 4$$
$$1 = 1 \text{ (which is true)}$$

Since $x = 2, -6$ satisfy the given equation, therefore, the solution set is $\{2, -6\}$ i.e. $S.S = \{2, -6\}$

(vi). $\frac{1}{2}|x+3| + 21 = 9$

Solution: As given $\frac{1}{2}|x+3| + 21 = 9$

$$\frac{1}{2}|x+3| = 9 - 21$$
$$\frac{1}{2}|x+3| = -12$$
$$|x+3| = -24$$

Which is not possible as modulus value is always non-negative.

(vii). $\left|\frac{3-5x}{4}\right| - \frac{1}{3} = \frac{2}{3}$

Sol: As given $\left|\frac{3-5x}{4}\right| - \frac{1}{3} = \frac{2}{3}$

$$\left|\frac{3-5x}{4}\right| = \frac{2}{3} + \frac{1}{3}$$
$$\left|\frac{3-5x}{4}\right| = \frac{2+1}{3}$$
$$\left|\frac{3-5x}{4}\right| = \frac{3}{3}$$
$$\left|\frac{3-5x}{4}\right| = 1$$
$$|3-5x| = 4$$

By definition, we have

$3-5x = 4$ or $3-5x = -4$

$3-4 = 5x$ or $3+4 = 5x$

$-1 = 5x$ or $7 = 5x$

$x = -\frac{1}{5}$ or $x = \frac{7}{5}$

Check:

$$\left|\frac{3-5x}{4}\right| - \frac{1}{3} = \frac{2}{3} \ldots(1)$$

Put $x = -\frac{1}{5}$, in (1)

$$\left|\frac{3-5\left(-\frac{1}{5}\right)}{4}\right|-\frac{1}{3}=\frac{2}{3}$$
$$\left|\frac{3+1}{4}\right|-\frac{1}{3}=\frac{2}{3}$$
$$\left|\frac{4}{4}\right|-\frac{1}{3}=\frac{2}{3}$$
$$|1|-\frac{1}{3}=\frac{2}{3}$$
$$1-\frac{1}{3}=\frac{2}{3}$$
$$\frac{3-1}{3}=\frac{2}{3}$$
$$\frac{2}{3}=\frac{2}{3}\text{(which is true)}$$

Put $x=\frac{7}{5}$, in (1)

$$\left|\frac{3-5\left(\frac{7}{5}\right)}{4}\right|-\frac{1}{3}=\frac{2}{3}$$
$$\left|\frac{3-7}{4}\right|-\frac{1}{3}=\frac{2}{3}$$
$$\left|\frac{-4}{4}\right|-\frac{1}{3}=\frac{2}{3}$$
$$|-1|-\frac{1}{3}=\frac{2}{3}$$
$$1-\frac{1}{3}=\frac{2}{3}$$
$$\frac{3-1}{3}=\frac{2}{3}$$
$$\frac{2}{3}=\frac{2}{3}\text{(which is true)}$$

Since $x=-\frac{1}{5},\frac{7}{5}$ satisfy the given equation, therefore, the solution set is $\left\{-\frac{1}{5},\frac{7}{5}\right\}$ i.e. $S.S=\left\{-\frac{1}{5},\frac{7}{5}\right\}$

(viii). $\left|\frac{x+5}{2-x}\right|=6$

Solution: As given $\left|\frac{x+5}{2-x}\right|=6$

By definition, we have

$\frac{x+5}{2-x}=6$ or $\frac{x+5}{2-x}=-6$

$x+5=12-6x$ or $x+5=-12+6x$

$x+6x=12-5$ or $5+12=6x-x$

$7x=7$ or $17=5x$

$x=1$ or $x=\frac{17}{5}$

Check:

$$\left|\frac{x+5}{2-x}\right|=6\ldots(1)$$

Put $x=1$, in (1)

$$\left|\frac{1+5}{2-1}\right|=6$$
$$\left|\frac{6}{1}\right|=6$$
$$|6|=6$$
$$6=6\text{ (which is true)}$$

Put $x=\frac{17}{5}$, in (1)

$$\left|\frac{\left(\frac{17}{5}\right)+5}{2-\left(\frac{17}{5}\right)}\right|=6$$
$$\left|\frac{\frac{17+25}{5}}{\frac{10-17}{5}}\right|=6$$
$$\left|\frac{\frac{42}{5}}{\frac{-7}{5}}\right|=6$$
$$\left|\frac{42}{-7}\right|=6$$
$$|-6|=6$$
$$6=6\text{ (which is true)}$$

Since $x=1,\frac{17}{5}$ satisfy the given equation, therefore, the solution set is $\left\{1,\frac{17}{5}\right\}$ i.e. $S.S=\left\{1,\frac{17}{5}\right\}$

Absolute Value

A linear inequality in one variable x is an inequality in which the variable x occurs only to the first power and has the standard form

$$ax + b < 0, a \neq 0$$

where a and b are real numbers. We may replace the symbol $<$ by $>, \leq, \geq$ also.

EXERCISE 7.3

Q#1) Solve the following inequalities.

(i). $3x + 1 < 5x - 4$

Solution: As given $3x + 1 < 5x - 4$

$\Rightarrow\ 5 < 2x$

$\Rightarrow \dfrac{5}{2} < x$

Hence, $S.S = \{x | x > \frac{5}{2}\}$

(ii). $4x - 10.3 \leq 21x - 1.8$

Solution: As given $4x - 10.3 \leq 21x - 1.8$

$\Rightarrow\ -10.3 + 1.8 \leq 21x - 4x$

$\Rightarrow\ -8.5 \leq 17x$

$\Rightarrow\ -\dfrac{8.5}{15} \leq x$

$\Rightarrow\ x \geq -0.5$

Hence, $S.S = \{x | x \geq -0.5\}$

(iii). $4 - \frac{1}{2}x \geq -7 + \frac{1}{4}x$

Solution: As given $4 - \frac{1}{2}x \geq -7 + \frac{1}{4}x$

Multiply by 4

$\Rightarrow\ 16 - 2x \geq -28 + x$

$\Rightarrow\ 16 + 28 \geq\ x + 2x$

$\Rightarrow\ 44 \geq 3x$

$\Rightarrow\ x \leq \dfrac{44}{3}$

Hence, $S.S = \{x | x \leq \frac{44}{3}\}$

(iv). $x - 2(5 - 2x) \geq 6x - 3\frac{1}{2}$

Solution: As given $x - 2(5 - 2x) \geq 6x - 3\frac{1}{2}$

$x - 10 + 4x \geq 6x - \dfrac{7}{2}$

$5x - 10 \geq 6x - \dfrac{7}{2}$

Multiply by 2

$\Rightarrow\ 10x - 20 \geq 12x - 7$

$\Rightarrow\ -20 + 7 \geq\ 12x - 10x$

$\Rightarrow\ -13 \geq 2x$

$\Rightarrow\ x \leq \dfrac{-13}{2}$

$\Rightarrow\ x \leq -6.5$

Hence, $S.S = \{x | x \leq -6.5\}$

(v). $\frac{3x+2}{9} - \frac{2x+1}{3} > -1$

Sol: As given $\frac{3x+2}{9} - \frac{2x+1}{3} > -1$

Multiply by 4 (LCM), we have

$\Rightarrow 9 \times \left(\dfrac{3x + 2}{9}\right) - 9 \times \left(\dfrac{2x + 1}{3}\right) > 9 \times (-1)$

$\Rightarrow (3x + 2) - 3(2x + 1) > -9$

$\Rightarrow\ 3x + 2 - 6x - 3 > -9$

$\Rightarrow\ -3x - 1 > -9$

$\Rightarrow\ -1 + 9 > 3x$

$\Rightarrow\ 8 > 3x$

$\Rightarrow \dfrac{8}{3} > x$

Hence, $S.S = \{x | x < \frac{8}{3}\}$

(vi). $3(2x + 1) - 2(2x + 5) < 5(3x - 2)$

Solution: As given $3(2x + 1) - 2(2x + 5) < 5(3x - 2)$

$\Rightarrow\ 6x + 3 - 4x - 10 < 15x - 10$

$\Rightarrow\ 2x - 7 < 15x - 10$

$\Rightarrow\ -7 + 10 < 15x - 2x$

$\Rightarrow\ 3 < 13x$

$\Rightarrow \dfrac{3}{13} < x$

Hence, $S.S = \{x | x > \frac{3}{13}\}$

(vii). $3(x - 1) - (x - 2) > -2(x + 4)$

Solution: As given $3(x - 1) - (x - 2) > -2(x + 4)$

$\Rightarrow\ 3x - 3 - x + 2 > -2x - 8$

$\Rightarrow\ 2x - 1 > -2x - 8$

$\Rightarrow\ 2x + 2x > -8 + 1$

$\Rightarrow\ 4x > -7$

$\Rightarrow x > -\dfrac{7}{4}$

Hence, $S.S = \{x | x > -\frac{7}{4}\}$

(viii). $2\frac{2}{3} + \frac{2}{3}(5x - 4) > -\frac{1}{3}(8x + 7)$

Solution: As given $2\frac{2}{3} + \frac{2}{3}(5x - 4) > -\frac{1}{3}(8x + 7)$

$\Rightarrow\ \dfrac{8}{3} + \dfrac{2}{3}(5x - 4) > -\dfrac{1}{3}(8x + 7)$

Multiply by 3 (LCM)

$$\Rightarrow\ 3 \times \left(\frac{8}{3}\right) + 3 \times \left(\frac{2}{3}(5x - 4)\right) > 3 \times \left(-\frac{1}{3}(8x + 7)\right)$$

$\Rightarrow\ 8 + 2(5x - 4) > -(8x + 7)$

$\Rightarrow\ 8 + 10x - 8 > -8x - 7)$

$\Rightarrow\ 10x > -8x - 7$

$\Rightarrow\ 10x + 8x > -7$

$\Rightarrow\ 18x > -7$

$\Rightarrow\ x < -\dfrac{7}{18}$

Hence, $S.S = \{x | x < -\frac{7}{18}\}$

Q#2) Solve the following inequalities.

(i). $-4 < 3x + 5 < 8$

Solution: As given $-4 < 3x + 5 < 8$

$\Rightarrow\ -4 - 5 < 3x < 8 - 5$

$\Rightarrow\ -9 < 3x < 3$

$\Rightarrow\ -\dfrac{9}{3} < \dfrac{3x}{3} < \dfrac{3}{3}$

$\Rightarrow\ -3 < x < 1$

Hence, $S.S = \{x | -3 < x < 1\}$

(ii). $-5 < \frac{4-3x}{2} < 1$

Solution: As given $-5 < \frac{4-3x}{2} < 1$

Multiply by 2

$\Rightarrow -10 < 4 - 3x < 2$

$\Rightarrow 10 - 4 < 4 - 3x - 4 < 2 - 4$

$\Rightarrow -14 < -3x < -2$

Multiply by -1 (inequality changes)

$\Rightarrow 14 > 3x > 2$

$\Rightarrow \frac{14}{3} > x > \frac{2}{3}$

Hence, $S.S = \{x| \frac{14}{3} > x > \frac{2}{3}$

(iii). $-6 < \frac{x-2}{4} < 6$

Solution: As given $-6 < \frac{x-2}{4} < 6$

$\Rightarrow -24 < x - 2 < 24$

$\Rightarrow -24 + 2 < x - 2 + 2 < 24 + 2$

$\Rightarrow -22 < x < 26$

Hence, $S.S = \{x| -22 < x < 26\}$

(iv). $3 \geq \frac{7-x}{2} \geq 1$

Solution: As given $3 \geq \frac{7-x}{2} \geq 1$

$\Rightarrow 6 \geq 7 - x \geq 2$

$\Rightarrow 6 - 7 \geq -x \geq 2 - 7$

$\Rightarrow -1 \geq -x \geq -5$

Multiply by -1

$\Rightarrow 1 \leq x \leq 5$

Hence, $S.S = \{x|1 \leq x \leq 5\}$

(v). $3x - 10 \leq 5 < x + 3$

Sol: As given $3x - 10 \leq 5 < x + 3$

$3x - 10 \leq 5 \quad or \quad 5 < x + 3$

$\Rightarrow 3x \leq 5 + 10 \quad or \quad 5 - 3 < x$

$\Rightarrow 3x \leq 15 \quad or \quad 2 < x$

$\Rightarrow x \leq 5 \quad or \quad 2 < x$

$\Rightarrow 2 < x \quad or \quad x \leq 5$

$\Rightarrow 2 < x \leq 5$

Hence, $S.S = \{x|2 < x \leq 5\}$

(vi). $-3 < \frac{x-4}{-5} < 4$

Solution: As given $-3 < \frac{x-4}{-5} < 4$

Multiply by -5

$\Rightarrow -5 \times (-3) > -5 \times \left(\frac{x-4}{-5}\right) < -5 \times (4)$

$\Rightarrow 15 < x - 4 < -20$

$\Rightarrow 15 + 4 < x < -20 + 4$

$\Rightarrow 19 > x > -16$

Hence, $S.S = \{x| -16 < x < 19\}$

(vii). $1 - 2x < 5 - x < 25 - 6x$

Solution : As given $1 - 2x < 5 - x < 25 - 6x$

$1 - 2x < 5 - x \quad or \quad 5 - x < 25 - 6x$

$\Rightarrow 1 - 5 < -x + 2x \quad or \quad -x + 6x < 25 - 5$

$\Rightarrow -4 < x \quad or \quad 5x < 20$

$\Rightarrow -4 < x \quad or \quad x < 4$

$\Rightarrow -4 < x < 4$

Hence, $S.S = \{x| -4 < x < 4\}$

(viii). $3x - 2 < 2x + 1 < 4x + 17$

Solution: As given $3x - 2 < 2x + 1 < 4x + 17$

$3x - 2 < 2x + 1 \quad or \quad 2x + 1 < 4x + 17$

$\Rightarrow 3x - 2x < 1 + 2 \quad or \quad 1 - 17 < 4x - 2x$

$\Rightarrow x < 3 \quad or \quad -16 < 2x$

$\Rightarrow x < 3 \quad or \quad -8 < x$

$\Rightarrow -8 < x \; or \; x < 3$

$\Rightarrow -8 < x < 3$

Hence, $S.S = \{x| -8 < x < 3\}$

Complied by Shumaila Amin